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OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: **MAT 304**

COURSE TITLE: **LINEAR ALGEBRA II**

DATE: 5th DEC 2019 **TIME: 9AM-12PM**

INSTRUCTION TO CANDIDATES

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MAT 304: LINEAR ALGEBRA II

STREAM: BSc (CS&ASC) **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***

ii. *Do not write on the question paper.*

SECTION A – ATTEMPT ALL QUESTIONS IN THIS SECTION.

QUESTION ONE (16 MARKS)

$\sum_{j=1}^{n}$

a) Define the following terms

- i) Normalized vector (1 mark)
- ii) Inner product (1 mark)
- iii) Geometric multiplicity (1 mark)

b) Show that the $n-$space $(\Re^n)$ which is the inner product is defined as $<u,v>= u_1v_1 + u_2v_2 + ... + u_nv_n$ where $u = u_1 + u_2 + ... + u_n$ and $v = v_1 + v_2 + ... + v_n$ in an inner product space. (7 marks)

c) Verify that the set $S = \left\{(0,1,0), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\right\}$ is orthonormal in $\Re^3$.

(4 marks)

d) Show that $A = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ is not diagonalizable. (2 marks)

QUESTION TWO (15 MARKS)

a) Find the matrix P which diagonalizes the matrix $\begin{bmatrix} 3 & 2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (6 marks)

b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ by use of cayley-Hamilton theorem

(4 marks)

c) Prove that the minimum polynomial $m(a)$ of a matrix A divides every polynomial that has A as zero. (5 marks)

SECTION B: ATTEMPT ANY THREE QUESTIONS (39 MARKS)

QUESTION THREE (13 MARKS)

Identify the surface which is represented by the following quadratic equation, by first putting it in a standard conic form $3x^2 + 3y^2 + 5z^2 - 4xy = 45$ and hence sketch it. (13 marks)

QUESTION FOUR (13 MARKS)

Given that $T : \Re^2 \to \Re^2$ is a linear transformation defined as $T(x_1, x_2) = (x_1 + x_2, -2x_1 + 4x_2)$ and A is a matrix of T with respect to the basis $B\{(1,0),(0,1)\}$. Suppose A' is a matrix of T with respect to the basis $B'\{(1,1),(1,2)\}$.

a) Find the transitional matrix P from B' to B, (4 marks)

b) Find P^{-1}, (2 marks)

c) Obtain the matrix of representation A with respect to B, and (5 marks)

d) A' from A.. (2 marks)

QUESTION FIVE (13 MARKS)

a) If u and v are non-zero orthogonal vectors in an inner product space V, then prove that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$. (4 marks)

b) Verify the Euclidean inner product space for $u = (1,-1,2)$, $v = (0,2,0)$. (2 marks)

c) Verify the Cauchy-Schwartz inequality for vectors $u = (2,-3,)$, $v = (4,-6)$ in $\Re^2$ (4 marks)

d) If $S = \{u_1, u_2, \ldots u_n\}$ is an orthonormal set in an inner product space, V then show that S is linearly independent. (3 marks)

QUESTION SIX (13 MARKS)

a) Given that $T : \Re^2 \to \Re^2$ defined as $T(x_1, x_2) = (-x_2, x_1)$, Show that the distance between $T(u)$ and $T(v) \Rightarrow d(T(u), T(v)) = d(u, v)$. (5 marks)

b) Given that $f(t) = t + 3$, and $g(t) = t^2 + 3t + 3$ in the polynomial space P_2 is an inner product defined by $< f(t), g(t) >= \int_0^1 f(t)g(t)dt$. Find

i) $< f(t), g(t) >$ (4 marks)

ii) $\|g(t)\|$ and $\|f(t)\|^2$ (4 marks)

QUESTION SEVEN (13 MARKS)

Consider the matrix

$A =$
$B = PA'P'$

$$\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$$

a)	Write down its charestarestic polynomial,	(2 marks)
b)	Write down its charestarestic equation,	(2 mark)
c)	Find its eigen values,	(3 marks)
d)	Find the eigen vectors corresponding to each eigen value,	(4 marks)
e)	Find the basis for each eigen space.	(2 marks)

$(1-\lambda)(-1-\lambda)$

$-1 \cancel{-\lambda} \cancel{+\lambda} + \lambda^2$

$-1 + \lambda^2 - 4 = 0$

$\langle u, v \rangle \geq 0$

$U \cdot V = \|u\| \|v\| \cos\theta$ where $\theta \geq 90°$

$\|u\| = \sum_{i=1}^{n} u_i^2 \Rightarrow u_i^2 \geq 0$

Hence

$3/2 \, ^{+2}$

$3/2 \, ^{+2}$