ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge...
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

SECOND/THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE CS/ASC

COURSE CODE: MAT 216/310

COURSE TITLE: REAL ANALYSIS I

DATE: 4th DEC 2019 **TIME: 9AM-12PM**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

MAT 216/310: REAL ANALYSIS I

STREAM: BSc (CS&ASC) **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***

ii. *Do not write on the question paper.*

SECTION A – ATTEMPT ALL QUESTIONS IN THIS SECTION.

QUESTION ONE (16 MARKS)

a) Define the term real analysis and state areas in which it is used. (2 Marks)

b) Prove that $\sqrt{6}$ is not a rational number. (4 Marks)

c) If a, b and c are real numbers and $a+b=a+c$ then $b=c$. Prove. (4 Marks)

d) Show that the sequence $\left\{\frac{1}{n}\right\}$ is convergent to 0. (3 Marks)

e) Prove the theorem that if a and b are real numbers such that for any other positive real number ε, $a \leq b+\varepsilon$, then $a \leq b$. (3 Marks)

QUESTION TWO (15 MARKS)

a) Define the completeness axioms of real numbers. (4 Marks)

b) State the supremum and infimum of the following sets;

i) $S=\{1,2,3\}$ (3 Marks)

ii) $S=\{x: 2 \leq x<4\}$ (3 Marks)

c) Suppose S is a non-empty subject of $\Re$ with upper bound and $c<0$ prove that $\sup(c.s)=c.\sup S$. (5 marks)

SECTION B : ATTEMPT ANY THREE QUESTIONS (39 MARKS)

QUESTION THREE (13 MARKS)

a) Let (X, d) be a metric space and let $K>0$ then (x, d_x) is a metric space where $d_k(x, y)=Kd(x, y)$. State the conditions that make it a metric space. (5 Marks)

b) Evaluate $\lim\limits_{n \to \infty} \frac{2n^3-3n}{5n^3-4n^2-2}$ (5 Marks)

c) Give three examples of monotone sequence and state whether it is strictly increasing, decreasing or alternating sequences. (3 Marks)

QUESTION FOUR (13 MARKS)

a) Prove that $\lim_{n\to\infty}\left(\frac{n^2-1}{n^2+1}\right)=1$ (6 Marks)

b) Define the term convergence of a sequence. (2 Marks)

c) Prove that any Cauchy sequence is bounded. (5 Marks)

QUESTION FIVE (13 MARKS)

a) Show that $x^{16}+x^7-1=0$ has a solution $\alpha \in (0,1)$. (5 Marks)

b) Show that $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right)=0$. (5 Marks)

c) Define a Cauchy sequence . (3 Marks)

QUESTION SIX (13 MARKS)

a) State the upper and lower bound of sets:

i) $S=\{x:1\le x<3\}$ (2 Marks)

ii) $Y=\{x:x<0\}$ (2 Marks)

b) Prove that the set S of real numbers is bounded if and only if there exist a real number K such that $|x|\le K$, for all $x\in S$. (4 Marks)

c) Let $\{X_n\}$ be defined by $X_n=\frac{1}{2}\left(X_{n-1}+\frac{2}{X_{n-1}}\right)$, $n=2,3,4....$ such that $X_1=2$. Write down the sequence whose n^{th} term is X_n. (5 Marks)

QUESTION SEVEN (13 MARKS)

a) Show that $\sqrt{2}$ is an irrational number using the contradiction method. (5 Marks)

b) Let $X=\Re$ and d be defined as $d(x,y)=|x-y|$, show that $(\Re,d)$ is a metric space. (6 Marks)

c) Define a sequence. (2 Marks)