



OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

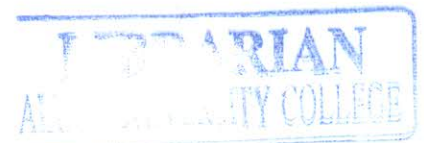
2019 /2020 ACADEMIC YEAR

SECOND/THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE CS/ASC

COURSE CODE: MAT 216/310

COURSE TITLE: REAL ANALYSIS I



DATE: 4th DEC 2019

TIME: 9AM-12PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES

PLEASE TURN OVER

MAT 216/310: REAL ANALYSIS I

STREAM: BSc (CS&ASC)

DURATION: 3 Hours

INSTRUCTION TO CANDIDATES

- i. Answer **ALL** questions from **section A** and any **THREE** from **section B**
- ii. Do not write on the question paper.

SECTION A – ATTEMPT ALL QUESTIONS IN THIS SECTION.**QUESTION ONE (16 MARKS)**

- a) Define the term real analysis and state areas in which it is used. (2 Marks)
- b) Prove that $\sqrt{6}$ is not a rational number. (4 Marks)
- c) If a, b and c are real numbers and $a + b = a + c$ then $b = c$. Prove. (4 Marks)
- d) Show that the sequence $\left\{\frac{1}{n}\right\}$ is convergent to 0. (3 Marks)
- e) Prove the theorem that if a and b are real numbers such that for any other positive real number ε , $a \leq b + \varepsilon$, then $a \leq b$. (3 Marks)

QUESTION TWO (15 MARKS)

- a) Define the completeness axioms of real numbers. (4 Marks)
- b) State the supremum and infimum of the following sets;
 - i) $S = \{1, 2, 3\}$ (3 Marks)
 - ii) $S = \{x : 2 \leq x < 4\}$ (3 Marks)
- c) Suppose S is a non-empty subset of \mathbb{R} with upper bound and $c < 0$ prove that $\sup(c.S) = c.\sup S$. (5 marks)

SECTION B : ATTEMPT ANY THREE QUESTIONS (39 MARKS)**QUESTION THREE (13 MARKS)**

- a) Let (X, d) be a metric space and let $K > 0$ then (X, d_K) is a metric space where $d_K(x, y) = Kd(x, y)$. State the conditions that make it a metric space. (5 Marks)
- b) Evaluate $\lim_{n \rightarrow \infty} \frac{2n^3 - 3n}{5n^3 - 4n^2 - 2}$ (5 Marks)
- c) Give three examples of monotone sequence and state whether it is strictly increasing, decreasing or alternating sequences. (3 Marks)

**QUESTION FOUR (13 MARKS)**

- a) Prove that $\lim_{n \rightarrow \infty} \left(\frac{n^2 - 1}{n^2 + 1} \right) = 1$ (6 Marks)
- b) Define the term convergence of a sequence. (2 Marks)
- c) Prove that any Cauchy sequence is bounded. (5 Marks)

QUESTION FIVE (13 MARKS)

- a) Show that $x^{16} + x^7 - 1 = 0$ has a solution $\alpha \in (0,1)$. (5 Marks)
- b) Show that $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} \right) = 0$. (5 Marks)
- c) Define a Cauchy sequence . (3 Marks)

QUESTION SIX (13 MARKS)

- a) State the upper and lower bound of sets:
- i) $S = \{x : 1 \leq x < 3\}$ (2 Marks)
- ii) $Y = \{x : x < 0\}$ (2 Marks)
- b) Prove that the set S of real numbers is bounded if and only if there exist a real number K such that $|x| \leq K$, for all $x \in S$. (4 Marks)
- c) Let $\{X_n\}$ be defined by $X_n = \frac{1}{2} \left(X_{n-1} + \frac{2}{X_{n-1}} \right)$, $n = 2, 3, 4, \dots$ such that $X_1 = 2$. Write down the sequence whose n^{th} term is X_n . (5 Marks)

QUESTION SEVEN (13 MARKS)

- a) Show that $\sqrt{2}$ is an irrational number using the contradiction method. (5 Marks)
- b) Let $X = \mathbb{R}$ and d be defined as $d(x, y) = |x - y|$, show that (\mathbb{R}, d) is a metric space. (6 Marks)
- c) Define a sequence. (2 Marks)