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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 26/7/2021 **TIME: 0800 – 1100 HRS**

INSTRUCTION TO CANDIDATES

a. SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

MAT 214

RUGULAR – MAIN EXAMINATION

MAT 214: VECTOR ANALYSIS

STREAM: **EDS/EDA/EDBS** **TIME:** 3 HRS

EXAMINATION SESSION: JULY **YEAR:** 2020/2021

INSTRUCTIONS TO CANDIDATES

(i) *Answer all questions in section A (Compulsory)*
(ii) *Answer any other THREE questions in section B*
(iii) *Answers should be comprehensive, informative and neat.*

SECTION A (31 MARKS)

Question One (16 Marks)

a). Define the term a gradient of a scalar field ϕ **(2 Marks)**

b). Find the direction cosines of $\boldsymbol{r} = 3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}$. **(2 Marks)**

c). Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$. **(4 Marks)**

d). Evaluate $\iiint_v \boldsymbol{F}dV$ where V is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2, z = 4$ if $\boldsymbol{F} = 2xz\boldsymbol{i} - x\boldsymbol{j} + y^2\boldsymbol{k}$. **(4 Marks)**

e). Find the total work done in moving a particle in a force field given by $\boldsymbol{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. **(4 Marks)**

Question Two (15 Marks)

a). Using green's theorem evaluate $\oint_C (y - \sin x)dx + \cos x \ dx$ where C is the triangle whole vertices are $O(0,0)$, $A\left(\frac{\pi}{2}, 0\right)$ and $B\left(\frac{\pi}{2}, 1\right)$. **(6 Marks)**

b). A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the components of its velocity at $t = 1$ in the direction $\boldsymbol{v} = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$. **(5 Marks)**

c). Find the divergence of $\boldsymbol{v} = x^2z\boldsymbol{i} - 2y^3z^2\boldsymbol{j} + xy^2z\boldsymbol{k}$ at point $(1, -1, 1)$. **(4 Marks)**

SECTION B (39 MARKS)

Question Three (13 Marks)

Given the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$, find the

a). Unit tangent vector **(3 Marks)**

b). Curvature **(5 Marks)**

c). Torsion **(5 Marks)**

Question Four (13 Marks)

a).Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$. **(5 Marks)**

b).(i). Show that $\boldsymbol{F} = (2xy + z^3)\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\boldsymbol{k}$ is a conservative force field. **(2 Marks)**

(ii). Find the scalar potential and the work done in moving an object in the field from $(1, -2, 1)$ to $(3, 1, 4)$. **(6 Marks)**

Question Five (13 Marks)

a). Find the $\boldsymbol{curl}\ (\boldsymbol{curl}\ \boldsymbol{v})$ given that $\boldsymbol{v} = x^2y\boldsymbol{i} - 2xz\boldsymbol{j} + 2yz\boldsymbol{k}$. **(4 Marks)**

b). Evaluate $\int \int_S \boldsymbol{F} \cdot \boldsymbol{n}\, ds$ where $\boldsymbol{F} = 4xz\boldsymbol{i} - y^2\boldsymbol{j} + yz\boldsymbol{k}$ where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. **(9 Marks)**

Question Six (13 Marks)

a). If $u = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$ and $v = \boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$, find $\boldsymbol{u} \times \boldsymbol{v}$. **(2 Marks)**

b). Find the work done in moving a particle once around a circle C in the xy plane in a force field $\boldsymbol{F} = (2x - y + z)\boldsymbol{i} + (x + y - z^2)\boldsymbol{j} + (3x - 2y + 4z)\boldsymbol{k}$ if the circle has a center at the origin and radius **3**. **(6 Marks)**

c). Using stokes theorem evaluate $\iint_S (\nabla \times \boldsymbol{A})d\boldsymbol{S}$ where $\boldsymbol{A} = (x + y)\boldsymbol{i} + (2y - x)\boldsymbol{j} + z^2\boldsymbol{k}$ and S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$. **(5 Marks)**

Question Seven (13 Marks)

a). Find the projection of vector $\boldsymbol{A} = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ on vector $\boldsymbol{B} = 4\boldsymbol{i} - 4\boldsymbol{j} + 7\boldsymbol{k}$. **(3 Marks)**

b). Find the unit vectors to the surface $\boldsymbol{r} = r(u_1, u_2, u_3)$. **(2 Marks)**

c). Evaluate $\int \int_S (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{n}\, ds$ where $\boldsymbol{F} = y\boldsymbol{i} + (x - 2xz)\boldsymbol{j} - xy\boldsymbol{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above xy plane. **(8 Marks)**