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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: **STA 216**

COURSE TITLE: **MATHEMATICAL STATISTICS II**

DATE: 28/7/2021 **TIME: 0800-1100HRS**

INSTRUCTION TO CANDIDATES

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REGULAR – MAIN EXAM

STA 216: MATHEMATICAL STATISTICS II

STREAM: ASC **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

SECTION A [31 Marks]. Answer ALL questions.

QUESTION ONE [15 Marks]

a) Define the following terms as used in statistics [3 Marks]
   i) A random variable.
   ii) Test statistic.
   iii) Critical region.

b) Distinguish clearly between dependent and independent samples. [4 Marks]

c) State the central limit theorem. [3 Marks]

d) Let $X \sim U(a,b)$ and also define $y = 5x$. Find the destination of Y [3 Marks]

e) State **two** properties of multinomial experiments. [2 Marks]

QUESTION TWO [16 Marks]

a) Suppose the joint density of two random variables x and y given by;

$$f(x,y) = \begin{cases} \frac{1}{4}(x+4y) & 0 < x < 2, 0 < y < 1 \\ 0 & \text{Otherwise} \end{cases}$$

The marginal density of x and y are $f_x(x) = \frac{1}{4}(x+2)$ and $f_y(y) = \frac{1}{4}(2+8y)$ respectively. Find the conditional distribution of x given y and the probability $X \leq 1$ that given that $y = \frac{1}{4}$

[5 Marks]

b) Let X and Y be independent count random variables with probability generating functions $G_X(S)$ and $G_Y(S)$, and also let $Z = X + Y$.

Show that $G_Z(S) = G_X(S)G_Y(S)$ [3 Marks]

c) Let $X_1, X_2, ..., X_n$ be a random sample of size $n = 36$ from a population that has a mean $\mu = 82.76$ and variance $\sigma^2 = 67.72$. Let $\overline{X}$ be the sample mean. What is the probability that the sample mean is between 79.02 and 83.93? [3 Marks]

d) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4from a distribution having pdf $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & elsewhere \end{cases}$ Find $F_{Y_1}(y)$ [5 Marks]

SECTION B [39 Marks] Answer any THREE questions]

QUESTION THREE [13 Marks]

a) Define multivariate random variables for n -dimensional random vector whose components are continuous random variables. [2 Marks]

b) Let $X = (X_1, X_2, X_3, X_4)^T$ be a four-dimensional random vector with the joint pdf given by,

$$f_x = (x_1, x_2, x_3, x_4) = \frac{3}{4}(x_1^2 + x_2^2 + x_3^2 + x_4^2)I_x$$

Where $X = \{(x_1, x_2, x_3, x_4) : 0 < x_i < 1, i = 1,2,3,4\}$

Calculate the;

i) Expectation $E(X_1, X_2)$ [3 Marks]

ii) Marginal pdf (X_1, X_2) [3 Marks]

iii) Conditional pdf $f(x_3, x_4 \mid x_1 = \frac{1}{4}, x_2 = \frac{3}{4})$ [5 Marks]

QUESTION FOUR [13 Marks]

a) Define characteristic function of a random X and give any two properties. [4 Marks]

b) Suppose that X and Y are independent random variables. Let $W = X + Y$ show that for discrete random variable with pdfs $p_X(x)$ and $p_Y(y)$ $p_W(w) = \sum_{\text{all } x} p_X(x)p_W(w-x)$

[4 Marks]

c) Suppose X is a continuous random variable. Let $Y = aX + b$, where $a \neq 0$ and b is a constant. Show that, $f_Y(y) = \frac{1}{|a|} f_x\left(\frac{y-b}{a}\right)$ [5 Marks]

QUESTION FIVE [13 Marks]

a) Define Chebychev's inequality [3 Marks]

b) Let the probability density function of a random variable X be given by,

$$f(x) = \begin{cases} 70x^3(1-x)^3 & \text{if } 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

What is the exact value of $P(|X - \mu| \leq 4\sigma)$, and approximate value of $P(|X - \mu| \leq 4\sigma)$ using the Chebychev's inequality? [10 Marks]

QUESTION SIX [13 Marks]

a) Let $X_1, X_2, ..., X_k$ be identically and independently random variables with probability density function $f(X_i)$. Show that if $Y = \sum_{i=1}^{k} X_i$, then $M_y(t) = \prod_{i=1}^{k} M(t)$ [6 Marks]

b) Suppose X_1 and X_2 are identically and independently distributed Poisson (λ). Find the distribution of $Y = X_1 + X_2$ using the moment generating technique. [7 Marks]

QUESTION SEVEN [13 Marks]

Let X_1 follow $\chi^2(r_1)$ and X_2 follow $\chi^2(r_2)$. Further let X_1 and X_2 be independent. Find the distribution of $V = \frac{X_1/r_1}{X_2/r_2}$ [13 Marks]

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