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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

MAIN EXAM

COURSE CODE: MAT 311

COURSE TITLE: REAL ANALYSIS II

DATE: 18/03/2021 **TIME: 0900 – 1200 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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MAT 311

RUGULAR – MAIN EXAMINATION

MAT 311: REAL ANALYSIS II

STREAM: **BED SCI/ARTS** **TIME:** 3 HRS

EXAMINATION SESSION: MARCH **YEAR:** 2020/2021

INSTRUCTIONS TO CANDIDATES

(i) Answer all questions in section A (Compulsory)
(ii) Answer any other THREE questions in section B
(iii) Answers should be comprehensive, informative and neat.

SECTION A (31 MARKS)

Question One (16 Marks)

a). Define the following terms

i). A limit of a sequence **(1 Mark)**

ii). Lebesgue Measure **(2 Mark)**

iii). Uniform convergence of sequence of functions **(2 Marks)**

iv). Derivative of a functions f at a point p **(2 Marks)**

b). Describe the lower and upper Riemann integrals. **(4 Marks)**

c). Prove that if f is a function of bounded variation, then f is bounded. **(5 Marks)**

Question Two (15 Marks)

a). Let $\{x_n\}$ and $\{y_n\}$ be sequences which converge to x and y respectively. Prove that the sequence $\{x_n - y_n\}$ converge to $x - y$. **(3 Marks)**

b). Let $\{x_n\}$ and $\{y_n\}$ be defined as $x_n = \frac{1}{n}$ and $y_n = \frac{2+n}{n}$.

i). Determine the values of x and y if limits $x = \lim_{n\to\infty} x_n$ and $y = \lim_{n\to\infty} y_n$ **(2 Marks)**

ii). Compute $x_n + y_n$ and $x + y$ hence, show that the new sequence $\{x_n + y_n\}$ to $x + y$.

(4 Marks)

c). Prove that a composite function $g \circ f$ is continuous at a if f is continuous at a and g is continuous at $b = f(a)$. **(3 Marks)**

d). Find the limit of Convergence of the series $\sum_{n=0}^{\infty} \frac{1}{4^n}$. **(3 Marks)**

SECTION B (39 MARKS)

Question Three (13 Marks)

a). State and prove the Rolle's theorem. **(6 Marks)**

b). Show that the sequence $\{f_n\}$ defined by $f_n = \frac{nx}{1+n^2x^2}$ on $(0, \infty)$ converges pointwise to 0.

(4 Marks)

c). Prove that the sequence $\{x_n\}$ where $x_n = \frac{1}{n^2}$ is a Cauchy sequence. **(3 Marks)**

Question Four (13 Marks)

a). Find the values of x for which the series $\sum_{n=0}^{\infty} (8x)^n$ converges, hence, state the radius of convergence. **(4 Marks)**

b). Prove that a sequence $\{f_n\}$ of bounded functions on a set $D \subseteq \mathbb{R}^n$ to $\mathbb{R}^m$ converges uniformly on D to a function f if and only if $\|f_n - f\| \to 0$. **(5 Marks)**

c). (i). Define a right-hand side limit of a function $f(x)$. **(1 Mark)**

(ii). Let f be a function defined as $f(x) = \begin{cases} 3 - x & \text{for } x \leq 1 \\ 2x^2 & \text{for } x > 1 \end{cases}$. Find $\lim_{x\to 1} f(x)$. **(3 Marks)**

Question Five (13 Marks)

a). Prove the a Cauchy sequence is bounded. **(4 Marks)** b). Let $\{f_n\}$ be a sequence of functions defined as $f_n = \frac{1}{n}\cos^2(nx)$. Show that $\{f_n\}$ converges uniformly to 0. **(4 Marks)**

c). Prove that any monotonic increasing function is a functions of bounded variation. **(5 Marks)**

Question Six (13 Marks)

a). Let $f: [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$$

Show that f is not Riemann integrable but Lebesgue integrable **(5 Marks)**

b). Let $\sum_{n\in\mathbb{N}} x_n$ be a series of elements of $\mathbb{R}$. Prove that the series converges in $\mathbb{R}$ if and only if for each real number $\epsilon > 0$, there is an $N(\epsilon) \in \mathbb{N}$ such that

$$\left|\sum_{k=n}^{m} x_k\right| < \epsilon \text{ for all } m \geq n \geq N(\epsilon).$$

(6 Marks)

c). Let $V_f[a,b]$ be the total variation of the function f. Show that the total variation of the function $f(x) = 2$ is zero. **(1 Marks)**

Question Seven (13 Marks)

a) Let $\{x_n\}$ be a sequence of real numbers which is monotone increasing, then the sequence converges if and only if it is bounded, in which case, its limit is $\sup\{x_n\}$. **(8 Marks)**

b). i). What is a power series? **(1 Mark)**

ii). Determine the radius of convergence and the interval of Convergence for the powers series

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{(-3)^n}.$$

(4 Marks)

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