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**OFFICE OF THE DEPUTY PRINCIPAL**

**ACADEMICS, STUDENT AFFAIRS AND RESEARCH**

**UNIVERSITY EXAMINATIONS**

**2020 /2021 ACADEMIC YEAR**

**FOURTH YEAR SECOND SEMESTER REGULAR EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE & BACHELOR OF EDUCATION ARTS**

**COURSE CODE:** **MAT 415**

**COURSE TITLE:** **DIFFERENTIAL GEOMETRY**

**DATE: 15/7/2021** **TIME: 0800-1100HRS**

**INSTRUCTION TO CANDIDATES**

- **SEE INSIDE**

**THIS PAPER CONSISTS OF 4 PRINTED PAGES** **PLEASE TURN OVER**

**REGULAR – MAIN EXAM**

**MAT 415: DIFFERENTIAL GEOMETRY**

**STREAM: EDS & EDA** **DURATION: 3 Hours**

**INSTRUCTION TO CANDIDATES**

i. Answer **ALL** questions from **Section A** and any Three from **Section B**

ii. Do not write on the question paper.

**SECTION A: 31 MARKS (COMPULSORY SECTION)**

**QUESTION ONE (16 MARKS)**

a) A space curve C is given by the parametric equation $X_1 = 3t - t^3, X_2 = 3t^2$, $X_3 = 3t + t^3$. Calculate the following at any point $P(x, y, z)$ on the curve;

i) Unit tangent, $\vec{T}$

ii) Curvature, $k$

iii) Unit principal normal, $\vec{N}$

iv) Unit binormal, $\vec{B}$

v) Torsion, $\tau$

vi) Radius of torsion, $\sigma$ (13 Marks)

b) Obtain the equation of the tangent line to the curve

$\vec{X} = e^t i + e^{-t} j + t^2 k$ at $t = 1$ (3 Marks)

**QUESTION TWO (15 MARKS)**

a) Given that a curve C is defined as $\vec{X} = \vec{X}(t)$ and is of class 2, show that for this curve, the curvature $k$can be given as $k = \left|\vec{X}' \times \vec{X}''\right| / \left|\vec{X}'\right|^3$ (6 Marks)

b) Define the following terms as used in the theory of curves

i) Osculating plane

ii) Normal plane

iii) Rectifying plane (3 Marks)

c) Obtain the equations of the osculating plane, the rectifying plane, and the normal plane through the point $P(0,0,9)$ of a curve defined as $\vec{X} = t\,i + \frac{t^2}{2} j + \frac{t^3}{2} k$ (6 Marks)

**SECTION B: (ATTEMPT ANY THREE QUESTIONS)**

**QUESTION THREE (13 MARKS)**

a) Show that the surface $\vec{X} = u\,i + v\,j + (u^2 + v^2)k$ is elliptic, hyperbolic and parabolic for $v > 0, v < 0$ and $v = 0$ respectively (9 Marks)

b) Let $\vec{X} = e^t \cos t\,i + e^t \sin t\,j + e^t k$ define a curve C, obtain the arc length on the curve C, between $0 \leq t \leq \pi$ (4 Marks)

**QUESTION FOUR (13 MARKS)**

a) Given the equation of a curve C as $\vec{X} = (1 + t)\,i - t^2 j + (1 + t^3)k$. Obtain the equation of its tangent line in parametric form and the normal plane to it at $t = 2$ (5 Marks)

b) Find the first fundamental form of the surface
$\vec{X} = (u + v)\vec{e_1} + (u - v)\vec{e_2} + \text{uv}\vec{e_3}$ (3 Marks)

c) Find the first and second fundamental forms of the surface
$\vec{X} = \{a(u + v), b(u - v), \text{uv}\}$ (5 Marks)

**QUESTION FIVE (13 MARKS)**

a) Find the normal curvature $k_n$ and the normal curvature vector $\vec{k_n}$ of the curve $u = t^2$, $v = t$ on the surface $\vec{X} = u\,i + v\,j + (u^2 + v^2)k$ at $t = 1$ (10 Marks)

b) Consider the helix $\vec{X}(t) = a\cos t\,i + a\sin t\,j + \text{bt}k$. Find its unit tangent vector (2 Marks)

c) What do you understand by the term umbilical point of a surface? (1 Mark)

**QUESTION SIX (13 MARKS)**

a) Given the right helicoid $\vec{X} = \{u\cos\varphi, u\sin\varphi, u\varphi\}$ where $u$ and $\varphi$ are parameters, find the
   i) second fundamental magnitudes and hence
   ii) second quadratic form of the right helicoid (9 Marks)

b) For the surface in (a) above, obtain also the first fundamental magnitudes, hence determine the Gaussian curvature of the surface (4 Marks)

**QUESTION SEVEN (13 MARKS)**

a) Define the following terms as used in curves
   i) Principal curvature
   ii) Principal directions (2 Marks)

b) Determine the Gaussian curvature of the torus
$\vec{X} = (b + a\sin\varphi)\cos\theta\,i + (b + a\sin\varphi)\sin\theta\,j + a\cos\varphi k$ (11 Marks)

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