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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: PHY 314

COURSE TITLE: QUANTUM MECHANICS 1

DATE: 09/03/2021 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

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REGULAR – MAIN EXAM

PHY 314: QUANTUM MECHANICS 1

STREAM: BED (Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer **TWO** questions in section **A** and any other **THREE** questions in section **B**.*

You may need to use the following constants

- *Planck's constant* $h = 6.625 \times 10^{-34}\ m^2kgs^{-1}$
- $\hbar = 1.054\ x\ 10^{-34}\ Js$
- *Mass of an electron,* $Me = 9.11 \times 10^{-31}\ Kg$
- *Mass of a proton,* $Mp = 1.67 \times 10^{-27}\ Kg$
- *Electronic charge,* $e = 1.6\ x\ 10^{-19}\ C$
- $1eV = 1.672 \times 10^{-19} J$

SECTION A (28 MARKS)

Question One (14 Marks)

a) Derive the deBroglie wave-particle duality equation. (4 Marks)
b) Explain the significance of Young's double slit experiment. (2 Marks)
c) Making reference to the probabilistic interpretation of Quantum Mechanics, explain the quantum wave function. (1 Marks)
d) State the time dependent Schrödinger's equation, hence give the meanings of all the symbols used in the equation. (4 Marks)
e) Define the following terms as used in quantum mechanics.

i. Free particle (1 Mark)
ii. Bound states (1 Mark)
iii. Tunneling (1 Mark)
iv.

Question Two (14 Marks)

a) Derive the free particle Schrödinger equation in one dimension. (4 Marks)

b) Consider a particle whose normalized wave function is $\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}xe^{-\alpha x} & x > 0 \\ 0 & x < 0 \end{cases}$

Determine the value of x for which the probability density peaks. (3 Marks)

c) By using the Heisenberg's uncertainty principle, determine the uncertainty product between position and linear momentum operators. (2 Marks)

d) An eigenfunction of the operator $\frac{d^2}{dx^2}$ is $\psi = e^{2x}$. Find the corresponding eigenvalue.

(3 Marks)

e) State any two postulates of Quantum Mechanics. (2 Marks)

SECTION B (42 MARKS)

Question Three (14 Marks)

a) Derive the expression for the Hamiltonian operator of a quantized harmonic oscillator in terms of the creation and annihilation operators $\hat{a}^{\dagger}$ and $\hat{a}$ respectively; hence determine the expectation value of this Hamiltonian operator in the number eigenstate $|n\rangle$

(7 Marks)

b) Calculate the angular frequency of a quantized harmonic oscillator whose ground state energy is 3.4 eV. (3 Marks)

c) By starting with the time-dependent Schrödinger equation, derive the time independent Schrödinger equation and the time evolution operator. (4 Marks)

Question Four (14 Marks)

a) For a time-independent Hamiltonian H, the time-dependent Schroedinger equation has the solution $\Psi(\vec{r},t)=\psi(\vec{r})e^{-\frac{i}{\hbar}Ht}$ where the position –dependent wave function $\psi(\vec{r})$ satisfies an eigenvalue equation $\hat{H}\psi(\vec{r})=E\psi(\vec{r})$. Determine the physical meaning of the constant quantity E. (4 Marks)

b) Show that the general wave function for a free particle in one-dimensional motion is given by $\psi(x) = a\cos\frac{1}{\hbar}(px + \hbar\theta)$ where the constants are to be defined explicitly in the derivation. (10 Marks)

Question Five (14 Marks)

a) The wave function of a two-state particle is obtained as a superposition $\Psi = c_1(t)\psi_1(\vec{r}) + c_2(t)\psi_2(\vec{r})$ where $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are orthonormal state functions with respective time-dependent probability amplitudes $c_1(t)$ and $c_2(t)$. Show that the state probability amplitudes satisfy the normalization condition $|c_1(t)|^2 + |c_2(t)|^2 = 1$ (5 Marks)

b) Determine the eigenvalues and corresponding eigenvectors of the following operator

$$\begin{pmatrix} -1 & 2 \\ 2 & 2 \end{pmatrix}$$

(9 Marks)

Question Six (14 Marks)

a) A particle of mass m is in the state $\psi(x,t) = Ae^{-a\left(\left(m\frac{x^2}{\hbar}\right)+it\right)}$ where A and a are positive real constants. Find the expression for A (5 Marks)

b) Calculate the uncertainty product $\Delta x \Delta p$, hence state whether it is consistent with the uncertainty principle.

You may use the following standard integrals: $\int_0^{\infty} e^{-bx^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{b}}$; $\int_0^{\infty} x^2 e^{-bx^2}dx = \frac{1}{4b}\sqrt{\frac{\pi}{b}}$

(9 Marks)

Question Seven (14 Marks)

a) Consider the hydrogen atom eigenfuction $\Psi_{432}(r,\theta,\phi)$ *what is the*
   (i) Total energy of an electron in this state in eV (3 Marks)
   (ii) Total orbital angular momentum. (2 Marks)

b) Obtain the Clebsh-Gordan coefficient for a system having $j_1 = 1$ and $j_2 = {}^1/_2$ (9 Marks)

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