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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE

COURSE CODE: MAT 304E

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: 10TH JUNE, 2022 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

REGULAR – MAIN EXAM

MAT 304E ORDINARY DIFFERENTIAL EQUATIONS II

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from **section A** and any **THREE** from **section B**.*

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions in this section.

Question One (16 Marks)

a) Solve the initial value problem $\begin{cases} y' = 2x(y-1) \\ y(1) = y_0 \end{cases}$. (7 Marks)

b) Consider the functions $\forall x \in \mathbb{R}$, $f(x) = x, g(x) = x^2$ and $h(x) = x^3$. Use the Wroskian to show that f, g, h are linearly independent. (5 Marks)

c) Write the general solution to $y'''(x) - 2y'(x) = 0$. (4 Marks)

Question Two (15 Marks)

a) What is an analytic function? (2 Marks)

b) Write the series solution around $x_0 = 0$ of the function $f(x) = \frac{e^x}{2x+1}$ and find the radius of convergence. (5 Marks)

c) Solve the Ordinary differential equation $y' + xy = x^2$ with $y(0) = y_0$. (5 Marks)

d) Let $L: C^n(A) \to C^0(A)$ be a linear operator. Show that $\forall \lambda, \mu \in \mathbb{R}: \forall y_1, y_2 \in C^n(A)$: (3 Marks)

$$L(\lambda y_1 + \mu y_2) = \lambda L(y_1) + \mu L(y_2)$$

SECTION B (39 Marks)

Answer any THREE questions.

Question Three (13 Marks)

a) Solve the ODE initial value problem (13 Marks)

$$x^3 y'''(x) + x^2 y''(x) - 2xy'(x) + 2y(x) = f(x), \quad \forall x \in [1, \infty)$$

Question Four (13 Marks)

a) Use proof by induction to show that given an $a \in \mathbb{R} - (-1)\mathbb{N}^*$ with $(-1)\mathbb{N}^* = \{-x|x \in \mathbb{N}^*\} = \{-1, -2, -3, \dots\}$, we have: (7 Marks)

$$\forall n \in \mathbb{N}^* : \prod_{k=1}^{n}(k + a) = \frac{\Gamma(n + 1 + a)}{\Gamma(a + 1)}$$

b) Write the series expansion of the function $f(x) = e^x \cos x$ and find the radius of convergence. (6 Marks)

Question Five (13 Marks)

a) Solve the linear ODE

$$y''(x) + \cos(x)y(x) = 0$$

with a series around $x = 0$. (13 Marks)

Question Six (13 Marks)

a) State the techniques for solving ODEs. (2 Marks)

b) Solve the initial value problem. (5 Marks)

$$\begin{cases} y' = y^2 \\ y(0) = y_0 \end{cases}$$

c) Solve the initial value problem (6 Marks)

$$\begin{cases} y''(x) + \omega^2 y(x) = 0 \\ y(0) = y_0 \wedge y'(0) = y_1 \end{cases}$$

Question Seven (13 Marks)

a) Find the general solution to the initial value problem (13 Marks)

$$\begin{cases} y''(x) - xy(x) = 0 \\ y(0) = a_0 \wedge y'(0) = a_1 \end{cases}$$

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