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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: STA 112

COURSE TITLE: INTRODUCTION TO PROBABILITY AND STATISTICS I

DATE: 07/06/2022 **TIME:** 2.00PM - 5.00PM

INSTRUCTION TO CANDIDATES

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THIS PAPER CONSISTS OF 4 PRINTED PAGES PLEASE TURN OVER

STA 112: INTRODUCTION TO PROBABILITY AND STATISTICS I

STREAM:ASC **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

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SECTION A [31 Marks]. Answer ALL questions.

QUESTION ONE [15 Marks]

a) Define a discrete random variable. [2 Marks]

b) Statetwo properties for probability mass function of a discrete variable [2 Marks]

c) Let$X$ be a continuous discrete random variable with probability distribution $P(X=x)$, show that $\mathrm{E}[px+q]=p\mu+q$ where p and q are arbitrary constants [3 Marks]

d) Give two random events where Poisson distribution applies [2 Marks]

e) Assume that a continuous random variable$X$ has a probability density function given by

$$f(x)=\begin{cases}\frac{1}{4}x & 0\le x\le 2\\ 0 & elsewhere\end{cases}$$

Obtain the cumulative density function of X and hence $P(X\ge\frac{2}{5})$ [4 Marks]

f) Write down the probability mass function a real Bernoulli distribution [2 Marks]

QUESTION TWO [16 Marks]

a) Suppose the probability mass function of a discrete random variable T is as follows for t and $P(T=t)$ respectively, $(-4,0.12)$, $(-3,0.28)$, $(-2,0.16)$, $(-1,0.24)$, $(0,k)$

Find the value of the constant k and hence $P(-4\le t<0)$ [3 Marks]

b) Assume that a continuous random variable X has a probability density function given by $f(x)=\begin{cases}\frac{2}{7}x & 0\le x\le 2\\ 0 & elsewhere\end{cases}$ find the mean and variance of X [4 Marks]

c) Suppose that a biased coin is tossed four times and that the probability of heads on any toss is 0.4. Let X denote the number of heads that come up, calculate $P(2<t\le 4)$ [3 Marks]

d) Suppose in busy shopping outlet there are 500 customers per eight-hour day in a check-out lane, what is the probability that there will be exactly 3 in line during any five-minute period? [3 Marks]

e) Suppose that $M(t)$ exists for any $k\in N$ show that $\left.\frac{d^r M(t)}{dt^r}\right|_{t=0}=\mu_r=\mathrm{E}(X^r)$ [3 Marks]

SECTION B [39 Marks] [Answer any THREE Questions]

QUESTION THREE [13 Marks]

a) Definemoment generating function of a random variable X [2 Marks]

b) Giventhat $X \sim Po(\lambda)$ random variable, find;

i) Moment generating function [4 Marks]

ii)Mean and variance of X whose probability mass function is given by

$$f(x)=\begin{cases}\frac{1}{6}(\frac{5}{6})^x & x=0,1,2,\ldots\\ 0 & elsewhere\end{cases}$$ [7 Marks]

QUESTION FOUR [13 Marks]

a) Give anythreecharacteristics of a binomial experiment [2 Marks]

b) The probability distribution function of Binomial distribution is given by

$$P(X=x)=\begin{cases}\binom{n}{x}p^x q^{n-x} & x=0,1,2,\ldots,n\\ 0 & elsewhere\end{cases}$$

Use moment generating function technique to find its;

i) Moment generating function [4 Marks]

ii) Expectation and variance [7 Marks]

QUESTION FIVE [13 Marks]

a) Let X be a continuous random variable with probability density function

$$f(x)=\begin{cases}ax & 0\le x\le 1\\ a & 1\le x\le 2\\ 3a-ax & 2\le x\le 3\\ 0 & elsewhere\end{cases}$$

Determine the;

i) Constant a [3 Marks]

ii) $P(X \le 1.5)$ [4 Marks]

b) The time X, in hours between computer failures is a continuous random variable with density

b)

$$f(x)=\begin{cases}\lambda e^{-0.01x} & x>0\\ 0 & elsewhere\end{cases}$$

Find λ hence compute $P(50 \le X < 150)$ and $P(X < 100)$ [6 Marks]

QUESTION SIX [13 Marks]

a) Let X be a discrete random variable with probability mass function

$$P(X=x)=\begin{cases}\frac{x}{21} & x=1,2,3,4,5,6\\ 0 & elsewhere\end{cases}$$

Compute the mean of X [4 Marks]

b) Apacketcontaining 10 cards of which 8 have 2 red marks andthe other 2 have 5 black marks each. Let a person choose at random and without replacement 3cards from this packet and results represent sum of the resulting amounts. Find his expectation [5 Marks]

c) Computethemean number of points obtain in a single throw of an ordinary die[4 Marks]

QUESTION SEVEN [13 Marks]

A random variable X has a gamma distribution with probability distribution function given as

$$f(x)=\begin{cases}\frac{x^{\alpha-1}}{\Gamma\alpha\beta^{\alpha}}e^{\frac{-x}{\beta}} & 0<x<\infty\\ 0 & elsewhere\end{cases}$$

Find the moment generating function of X and hence mean and variance of X [13 Marks]

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