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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE

COURSE CODE: MAT 317

COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 10TH JUNE, 2022 **TIME: 0900 – 1200 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

Algorithm is a set

REGULAR – MAIN EXAM

MAT 317 NUMERICAL ANALYSIS I

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from* ***section A*** *and any* ***THREE*** *from* ***section B.***

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

c, p,

x^3+x-1

Answer ALL questions in this section.

Question One (16 Marks)

a) Using Newton-Raphson method, find a root near $x = 1$ of the equation $f(x) = x^3 + x - 1 = 0$. (4 Marks)

b) Compute a 4D-value of $\ln 9.2$ from $\ln 9.0 = 2.1972$, $\ln 9.5 = 2.2513$ by linear Lagrange interpolation and determine the error, using $\ln 9.2 = 2.2192$ (4D). (5 Marks)

c) Evaluate the integral $J = \int_0^1 e^{-x^2}\,dx$ by Gauss integration formula with $n = 3$. (5 Marks)

d) What is an error? (2 Marks)

Question Two (15 Marks)

a) i) Prove that in subtraction, a bound for the error of the results is given by the sum of the error bounds for the terms. (5 Marks)

ii) Prove that in multiplication, an error bound for the relative error of the results is given by the sum of the bounds for the relative errors of the given numbers. (5 Marks)

70

b) Round off the number 1.23454621 to

i) 2 decimals $x = 1.23454621$

$x_4 = 0.0000005$

ii) 5 decimals. (5 Marks)

SECTION B (39 Marks)

x^3+x-1

$(3x^2+1)=x$

$(3x^2+1)^3+3x+1-1$

Answer any THREE questions.

Question Three (13 Marks)

a) Find a solution of $f(x) = x^3 + x - 1 = 0$ by iteration. (6 Marks)

b) Find the positive solution of $2\sin x = x$. (4 Marks)

c) Find the positive solution of $f(x) = x - 2\sin x = 0$ by the secant method, starting from $x_0 = 2, x_1 = 1.9$. (3 Marks)

.0 0 0 0 0 5 10

1.23454621

1.23454621
0.005
1.23954621
Cosh 0.56
then trancate
that has both

1.23464621
$+\ 0.0000051$
1.23455121

Question Four (13 Marks)

a) Compute $\cosh 0.56$ from the Newton's forward difference interpolation formula and the four values in the following table and estimate the error. (8 Marks)

j	x_j	$f_j = \cosh x_j$	Δf_j	$\Delta^2 f_j$	$\Delta^3 f_j$
0	0.5	1.127626			
			0.057839		
1	0.6	1.185465		0.011865	
			0.069704		0.000677
2	0.7	1.255169		0.012562	
			0.082266		
3	0.8	1.337435			

b) Evaluate $J = \int_0^1 e^{-x^2} dx$ by means of the trapezoidal rule with $n = 10$. (5 Marks)

$\int_0^1 e^{-x^2} = \int_0^1 \frac{e^{-x^{2+1}}}{-x^{3+1}} - \frac{e^{-(1)^{2+1}}}{-(1)^{3+1}} = \frac{e^{-(1)^3}}{e(1)^3} > e^{-}$

Question Five (13 Marks)

a) What is an algorithm? (2 Marks)

b) Integrate $f(x) = \frac{1}{4}\pi x^4 \cos\frac{1}{4}\pi x$ from 0 to 2 with $n = 1$ apply the error estimation for Simpson's rule by halving h. (6 Marks)

c) Set up a Newton iteration for computing the square root, x of a given positive number c and apply it to $c = 2$. (5 Marks)

Question Six (13 Marks)

a) Compute a $7D$-value of the Bessel function $J_0(x)$ for $x = 1.72$ from the four values in the following table, using

i) Newton's forward formula (4 Marks)

ii) Newton's backward formula (4 Marks)

j_{for}	j_{back}	x_j	$J_0(x_j)$	$1st\ Diff$	$2nd\ Diff$	$3rd\ Diff$
0	−3	1.7	0.3979839			
				−0.0579985		
1	−2	1.8	0.3399864		−0.0001693	
				−0.0581678		0.0004093
2	−1	1.9	0.2818186		0.0002400	
				−0.0579278		
3	0	2.0	0.2238908			

b) Derive the trapezoidal rule. (5 Marks)

Question Seven (13 Marks)

a) Interpolate $f(x) = x^4$ on the interval $-1 \leq x \leq 1$ by the cubic spline $g(x)$ corresponding to the nodes $x_0 = -1, x_1 = 0, x_2 = 1$ and satisfying te damped conditions $g'(-1) = f'(-1),\ g'(1) = f'(1)$. (11 Marks)

b) Define the following terms:
   i) Round off error
   ii) Relative error. (2 Marks)

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