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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 9TH JUNE, 2022 TIME: 1400 – 1700 HRS

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

REGULAR – MAIN EXAM

MAT 214 VECTOR ANALYSIS

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from* ***section A*** *and any* ***THREE*** *from* ***section B***.

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions in this section.

Question One (16 Marks)

a) Find a unit vector parallel to the resultant of vectors $\boldsymbol{r_1} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, $\boldsymbol{r_2} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. (5 Marks)

b) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

i) Determine its velocity and acceleration at any time. (3 Marks)

ii) Find the magnitudes of the velocity and acceleration at $t = 0$. (3 Marks)

c) If $\vec{A}$ has constant magnitude, show that $\vec{A}$ and $\frac{d\vec{A}}{dt}$ are perpendicular provided $\left|\frac{d\vec{A}}{dt}\right| \neq 0$. (5 Marks)

Question Two (15 Marks)

a) If $\phi(x, y, z) = 3x^2y - y^3z^2$ find $\nabla\phi$ at the point $(1, -2, -1)$. (4 Marks)

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (6 Marks)

c) Find the total work done in moving in a force field given by $\vec{F} = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. (5 Marks)

SECTION B (39 Marks)

Answer any THREE questions.

Question Three (13 Marks)

a) Evaluate $\iint_S \vec{A}.\boldsymbol{n}\, dS$, where $\vec{A} = z\mathbf{i} + x\mathbf{j} - 3y^2z\mathbf{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. (8 Marks)

b) Find the work done in moving a particle once around a circle C in the xy plane, if the circle has center at the origin and radius 3 and if the force field is given by (5 Marks)

$$\vec{F} = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$$

Question Four (13 Marks)

a) Find the $curl(rf(r))$ where $f(r)$ is differentiable. (5 Marks)

b) Find an equation for the plane determined by the points $P_1(2,-1,1), P_2(3,2,-1)$ and $P_3(-1,3,2)$. (4 Marks)

c) Determine a unit vector perpendicular to the plane of $\vec{A} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ and $\vec{B} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. (4 Marks)

Question Five (13 Marks)

a) A man travelling southward at 15 $mileshr^{-1}$ observes that the wind appears to be coming from the west. On increasing his speed to 25 $mileshr^{-1}$ it appears to be coming from the southwest. Find the direction and speed of the wind. (4 Marks)

b) Determine the vector having initial point $P(x_1, y_1, z_1)$ and terminal point $Q(x_2, y_2, z_2)$ and find its magnitude. (4 Marks)

c) An airplane moves in a northwest direction at $125 mileshr^{-1}$ relative to the ground, due to the fact there is a west wind of 50 $mileshr^{-1}$ relative to the ground. How fast and in what direction would the plane have traveled if there were no wind? (2 Marks)

d) Given $\boldsymbol{r}_1 = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\boldsymbol{r}_2 = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$, $\boldsymbol{r}_3 = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the magnitudes of $2\boldsymbol{r}_1 - 3\boldsymbol{r}_2 - 5\boldsymbol{r}_3$. (3 Marks)

Question Six (13 Marks)

a) Find the projection of the vector $\vec{A} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ on the vector $\vec{B} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$. (4 Marks)

b) If $R(u) = x(u)\mathbf{i} + y(u)\mathbf{j} + z(u)\mathbf{k}$, where x, y and z are differentiable functions of a scalar u, prove that $\frac{dR}{du} = \frac{dx}{du}\mathbf{i} + \frac{dy}{du}\mathbf{j} + \frac{dz}{du}\mathbf{k}$. (4 Marks)

c) A particle moves so that its position vector is given by $\boldsymbol{r} = \cos\omega t\mathbf{i} + \sin\omega t\,\mathbf{j}$ where ω is a constant. Show that:

i) The velocity $\vec{v}$ of the particle is perpendicular to r (2 Marks)

ii) The acceleration $\vec{a}$ is directed toward the origin and has magnitude proportional to the distance from the origin. (3 Marks)

Question Seven (13 Marks)

a) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$. (3 Marks)

b) If $\vec{A} = x^2z\mathbf{i} - 2y^3x^2\mathbf{j} + xy^2z\mathbf{k}$, find $\nabla.\vec{A}$ at the point $(1,-1,1)$. (4 Marks)

c) A fluid moves so that its velocity at any point is $v(x, y, z)$. Show that the loss of fluid per unit volume per unit time in a small parallel to the coordinate axes and having magnitude $\Delta x, \Delta y, \Delta z$ respectively, is given approximately by $div\ v = \nabla.v$. (6 Marks)