ALUPE UNIVERSITY
OFFICE OF THE DEPUTY VICE CHANCELLOR
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS
2023 /2024 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: MAT 113
COURSE TITLE: DIFFERENTIAL CALCULUS

DATE: 11ᵗʰ December 2023   **TIME: 2:00PM-5:00PM**

INSTRUCTION TO CANDIDATES
- SEE INSIDE

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MAT 113: DIFFERENTIAL CALCULUS

STREAM: Bed (Sc/Arts/Bus)   **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i.   Answer **ALL** questions from **section A** and any **THREE** from **section B**

ii.   Do not write on the question paper.

SECTION A (31 MARKS): Answer all questions in this section.

QUESTION ONE (16 MARKS)

a) Find $\dfrac{dy}{dx}$ for $x^3 + 3x^2 y + y^3 = 1$ (3 Marks)

b) Define composition of maps.   (2 Marks)

c) Find the nature of the turning points on the curve $y = x^3 - 3x^2 + 2$ (4 Marks)

d) Given that $f'(x) = 6x^2 + 6x - 4$ and $f(1) = 3$. Find $f(x)$ (4 Marks)

e) Evaluate $\dfrac{d}{dx}(\sinh x)$ (3 Marks)

QUESTION TWO (15 MARKS)

f) Differentiate $y = \sin^2 x$ (2 Marks)

g) Find velocity and acceleration when $t = 0$ given that $s = 12t - 4t^2$ (4 Marks)

h) Differentiate $y = x^x$ (2 Marks)

i) Find $\dfrac{dy}{dx}$ for $y = \dfrac{1}{\left(x^2 + 3x + 5\right)^3}$ (4 Marks)

j) Consider the function $f(x) = \begin{cases} 2x - 1 & , \quad x < 3 \\ 8 - x & , \quad 3 \le x \end{cases}$. Proof that f is continuous at x=3.

                                                                                 (3 Marks)

SECTION B: (39 MARKS)

QUESTION THREE (13 MARKS)

a) Differentiate

    i) $f(x) = (7 + x)^2$ (3 Marks)

    ii) $\sin^2 x \cos x$ with respect to x. (4 Marks)

b) Show that: $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x}$ is egual to zero. (4 Marks)

c) Evaluate $\sin^{-1}\left(\dfrac{1}{2}\right)$ (2 Marks)

QUESTION FOUR (13 MARKS)

a)   Given that: $f(x) = \dfrac{x+2}{x+1}$. Find all the values of c in the interval $(1, 2)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{where } a = 1 \text{ and } b = 2 \qquad \text{(5 Marks)}$$

b)   The distance s metres moved by a body in t seconds is given by

$s = 2t^3 - 12t^2 + 24t + 10$;Find the

    i) velocity when $t = 4$ (2 Marks)

    ii) value of t when the body comes to rest (3 Marks)

    iii) value of t when the acceleration is $10 m/s$ (3 Marks)

QUESTION FIVE (13 MARKS)

a) Differentiate the following with respect to x

    i) $y = x\sqrt{x} + 3$ (2 Marks)

    ii) $x \sin y = y \sin x$ (2 Marks)

    iii) $y = \ln \dfrac{x^2 + 1}{x^2 - 1}$ (2 Marks)

b) Find the tangent to the curve $y = 2x^3 - 4x^2 + 2$ at $(1,1)$ (3 Marks)

c) If $x = a(t^2 - 1), y = 2a(t + 1)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t (4 Marks)

QUESTION SIX: (13 MARKS)

a) Evaluate the limits

   i) $\lim\limits_{z \to 0} \dfrac{\sin(10z)}{z}$ (3 Marks)

   ii) $\lim\limits_{x \to 0} \dfrac{\sin(12x)}{\sin(5x)}$ (3 Marks)

b) Differentiate the following functions

   i) $y = \dfrac{2x + 3}{\sin 4x}$ (4 Marks)

   ii) $y = \ln(1 - 2x)^3$ (3 Marks)

QUESTION SEVEN (13 MARKS)

a)   A plane is flying directly away from you at 500 mph at an altitude of 3 miles. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you   (7 Marks)

b)   If $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x - 2)^2} & , \quad x \neq 2 \\ k & , \quad x = 2 \end{cases}$ is continuous at $x = 2$, find the value of k (6 Marks)