ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge....
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: STA112

COURSE TITLE: INTRODUCTION TO PROBABILITY AND STATISTICS I

DATE: 13TH OCTOBER, 2020 TIME: 2.00 PM – 5.00 PM

INSTRUCTION TO CANDIDATES

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THIS PAPER CONSISTS OF 4 PRINTED PAGES PLEASE TURN OVER

REGULAR – MAIN EXAM

STA 112: INTRODUCTION TO PROBABILITY AND STATISTICS I

STREAM: ASC **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and **ANY THREE** Questions in section B.
All questions in section B carry Equal Marks

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SECTION A (31 marks): Answer ALL questions.

QUESTION ONE (16 marks)

a) List and explain two main uses of poison distribution [2Marks]

b) A continuous random variable X has a pdf as follows

$$f(x) = \begin{cases} C(x^2 - 2x + 3) & \text{for } 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) C [2Marks]

ii) $\Pr(X \le 1)$ [2Marks]

c) Let X be a random variable of continuous type, defined by the pdf f(x)

$$f(x) = \begin{cases} \frac{2}{x^3} & 1 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Find the cumulative density function of the random variable X . [3Marks]

d) Let X be a random variable with probability density function f(x).

$$f(x) = \begin{cases} \frac{1}{18}(2x + 3)2 \le x \le 4 \\ 0, & \text{else where} \end{cases}$$

Obtain mean and variance of X [4Marks]

e) List three properties of expectation [3Marks]

QUESTION TWO (15 marks)

a) Consider the function $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0, \text{elsewhere} \end{cases}$

Obtain

i) the m.g.f. of X hence [4Marks]

ii) the mean and the variance of X [5Marks]

b) For every 100,000 employees, with a poison distribution we expect an accident rate of 2.2 per 10,000 employees. Find $P(x \geq 1)$ [3Marks]

c) If for a random variables $x_1, x_2\, x_3, \dots, x_n$ occur with frequencies $f_1, f_2, f_3, \dots, f_n$. Show that the second moment about the mean is given by $M_2 = M'_2 - (M'_1)^2$ [3Marks]

SECTION B (39 marks):

Answer any THREE questions. All Questions carry equal marks

QUESTION THREE (13 marks)

a) Given that a random variable X has the distribution with $f(x) = \begin{cases} \frac{1}{5-a}, & a < x < 5 \\ 0, & \text{elsewhere} \end{cases}$

i) Show that f(x) is a p.d.f. [3Marks]

ii) Find the mean and variance of X [6Marks]

b)) Given that a random variable X follows a Bernoulli distribution, obtain the first two raw moments. [4Marks]

QUESTION FOUR (13 marks)

a) An unbiased coin is thrown three times. If the random variable X denotes the number of heads obtained, find the cumulative density function of X. [4Marks]

b) If the masses of 300 students are normally distributed with mean 68kg and standard deviation 3kg. How many students have mass greater than 72kg [4Marks]

c) Let us assume telephone calls to a toll-free 800 number are made in accordance with the poison process assumption at the rate of 120 per hour during the period 9 A.M to 12 noon. Find the probability of at least one call being made during any given;

i) Minute [2Marks]

ii) Second [3Marks]

QUESTION FIVE (13 marks)

The probability distribution function of Poisson distribution with parameter λ is given by

$$f(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2\cdots \\ 0, & \text{elsewhere} \end{cases}$$

Find the moment generating function of X. Hence find its expectation and variance using the moment generating function [13 Marks]

QUESTION SIX (13 marks)

a) A college professor, based on his past experience, feels that there is a probability of 0.001 that he will be late to any given class and that being late or not for any class has no effect whether or not he is late for any other class. Then the number of times X that he will be late to his next 100 classes is a binomial random variable with parameters n=100 and p=0.001. Find the exact probabilities that he is late exactly 0 times and exactly 1 times.

[3Marks]

b) The probability distribution function of a beta distribution f(x) is given as;

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & 0 < x < 1 \quad \alpha, \beta > 0 \\ 0, \text{else where} & \end{cases}$$

Find

i) E(X) [4Marks]

ii) Var(x) [6Marks]

QUESTION SEVEN (13 marks)

a) If C is a constant, show that the m.g.f of cx is $M_X(ct)$ [3Marks]

b) Let X be a normal distribution random variable. i.e.

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} & -\infty < x < \infty \\ 0, & \text{else where} \end{cases}$$

Show that $f(x)$ is a p.d.f. [7Marks]

c) The variable X can assume the value r with probability function $\frac{1}{4}(\frac{3}{4})^r$ for r=0, 1, 2...
Show that X is a discrete random variable. [3Marks]

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