ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge....

P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, RESEARCH AND STUDENT AFFAIRS

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 212

COURSE TITLE: MATHEMATICAL STATISTICS I

DATE: 4th DEC 2019 **TIME: 9:00am-12:00noon**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 5 PRINTED PAGES **PLEASE TURN OVER**

STA 212: MATHEMATICAL STATISTICS I

STREAM: BSC (ASC) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. Answer **ALL** questions from section A and **ANY THREE** Questions in section B.

ii. Do not write on the question paper.

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SECTION A [31 MARKS]

QUESTION ONE (16 MARKS)

a) Define the term correlation [2Mks]

b) Let $f\left({}^{x1}/_{x2}\right) = \begin{cases} C_1 \frac{x_1}{x_2} & 0 < x_2 < 1 \quad 0 < x_1 < x_2 \\ 0 & elsewhere \end{cases}$

$f(x_2) = \begin{cases} C_2 {x_2}^4 & 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$

denote respectively the conditional p.d.f of x_1 given x_2 and the marginal p.d.f of x_2. Determine

i. The constants C_1 and C_2 [4Mks]

ii. The joint p.d.f of x_1 given x_2 [3Mks]

iii. $pr(\frac{1}{4} < x_1 < \frac{1}{2} | x_2 = \frac{5}{8})$ [3Mks]

iv. $pr(\frac{1}{4} < x_1 < \frac{1}{2})$ [2Mks]

c) Let the joint pdf of X given Y be defined by $f(x, y) = \begin{cases} \frac{x+y}{21} & x = 1,2 \quad y = 1,2,3 \\ 0 & elsewhere \end{cases}$

Obtain the marginal p.d.f of $f(x)$ [2Mks]

QUESTION TWO (15 MARKS)

Let the joint pdf of x given y be defined by $f(x_1, x_2) = \begin{cases} 2 & 0 < x_1 < x_2 < 1 \\ 0 & elsewhere \end{cases}$

Find the

a) Marginal density function of x_1 and x_2 [4Mks]

b) Conditional density of x_1 given x_2 [2Mks]

c) Conditional expectation mean of x_1 given x_2 [3Mks]

d) Conditional variance of x_1 given x_2 [3Mks]

e) Expected value of X and Y [3Mks]

SECTION B (39 MARKS)

QUESTION THREE (13 MARKS)

a) Given that $f(x,y) = \begin{cases} 2 & 0 < x < y, \quad 0 < y < 1 \\ 0 & otherwise \end{cases}$

Show that $E\left(y/_x\right) = \frac{1+x}{2}$ [3Mks]

b) The joint discrete probability density function of X and Y is given by;

x,y	(1,1)	(1,2)	(1,3)	(1,4)	(2,2)	(2,3)	2,4)	(3,3)	(3,4)	(4,4)
f(x,y)	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{4}{16}$

What is the

i) Density of Y given that $X = 2$ [2Mks]

ii) Y given that $X = 3$ [2Mks]

iii) X given that $Y = 2$ [2Mks]

vi) X given that Y=4 [2Mks]

c) Define the term stochastic independence. [2Mks]

QUESTION FOUR (13 MARKS)

a) Show that $f(x,y)$ is a p.d.f. [4Mks]

$$f(x,y) = \begin{cases} \dfrac{e^{-2\lambda}\lambda^{x+y}}{x!\ \ y!} & x,y = 0,1,2 \ldots \ldots. \\ 0 & elsewhere \end{cases}$$

b) From (a) above find the marginal probability density function for X [3Mks]

c) From (a) above find:

i) $f(x/y)$ [2Mks]

ii) $f(y/x)$ [2Mks]

d) If (X, Y) have bivariate cumulative distribution function

$$F(x,y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)}, \qquad x \geq 0, y \geq 0$$

Find the joint pdf of X and Y [2Mks]

QUESTION FIVE (13 MARKS)

Given that X and Y have joint density.

$$f(x,y) = \frac{1}{8}(6 - x - y)\ \ I_{(0,2)}(x),\ \ I_{(2,4)}(y),$$

Find

a) $E[Y/X = x]$ [3Mks]

b) $E(Y^2/X = x)$ [3Mks]

c) $Var[Y/X = x]$ [3Mks]

d) Show that $E[Y] = E[E[Y/X = x]$ [4Mks]

QUESTION SIX (13 MARKS)

a) If X and Y have joint probability density function $f(x, y)$ and marginal densities $f_X(x)$ and $f_Y(y)$ respectively. If $M_{XY}(t_1t_2)$ is the joint moment generating function of the distribution of X and Y. Prove that X and Y are stochastically independent iff $M_{XY}(t_1t_2) = M_X(t_1)M_Y(t_2)$ prove; [5Mks]

b) Given the joint probability distribution of X and Y as shown below;

Y	X	
	0	1
-1	0.125	0.5
0	0	0.25
1	0.125	0

Find

i) The marginal density of X [2Mks]

ii) The marginal density of Y [2Mks]

c) X and Y are two random variables with probability density

$$f(x, y) = \begin{cases} x + y & 0 < x < 1 \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find correlation coefficient of x and y [4Mks]

QUESTION SEVEN (13 MARKS)

a) If random variables X and Y have the following distribution

$$f(x, y) = \frac{n!}{x!\, y!\, (n - x - y)!} p^x q^y (1 - p - q)^{n-x-y}$$

for

$$x, y = 0,1,2, \dots n \;\; and \;\; x + y \leq n, 0 \leq p, 0 \leq q, p + q \leq 1$$

a. Obtain the moment generating function of (X,Y) [7Mks]

b. Using $M_{X,Y}(t_1, t_2)$ find E(x) and E(y) [6Mks]

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