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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION/ THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: **STA 216**

COURSE TITLE: **MATHEMATICAL STATISTICS II**

DATE: 28TH OCTOBER, 2020 **TIME: 0900 – 1200 HRS**

INSTRUCTION TO CANDIDATES

a) **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

REGULAR – MAIN EXAM

STA 216: MATHEMATICAL STATISTICS II

STREAM: ASC **DURATION: 3 hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

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SECTION A [31 Marks] Answer All questions]

QUESTION ONE [15 MARKS]

a) Define the following terms giving examples [6 marks]

a) Joint probability density function.

b) Stochastic independence.

c) Marginal density.

b) If X and Y have joint p.d.f $f(x, y)$ describe the conditional density of X given Y.

[3 Marks]

c) Let the joint p.d.f of x_1 and x_2 be $f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0\ elsewhere \end{cases}$

a) Find the marginal probability density function of x_1 and x_2. [3 marks]

b) Show that x_1 and x_2 are dependent. [3 marks]

QUESTION TWO [16 MARKS]

Let X, Y, and Z have a joint pdf $X, Y, Z \in \mathbb{R}^3$

$$f(X, Y, Z) = \frac{1}{K}, 0 < X < 1, 0 < Y < 1, 0 < Z < 1$$

Find the

a) Value of K [2 marks]
b) Marginal density functions $f(X)$, $f(Y)$ and $f(Z)$ [6 marks]
c) Conditional density function $f(X/YZ)$, $f(Z/XY)$ [6 marks]
d) Expectation E(Y/XY) [2 marks]

SECTION B [39 Marks] ANSWER ANY THREE QUESTIONS

QUESTION THREE [13 MARKS]

a) Describe any three properties of a characteristic function [6 Marks]

b) Let X_1 and X_2 be independent, with X_1 normal (0,1) and X_2 chi-square with r degrees of freedom. Show that the random variable $Y = \sqrt{r}\,X_1 \big/ \sqrt{X_2}$ has the T distribution with r degrees of freedom. [7 Marks]

QUESTION FOUR [13 MARKS]

Let X_1, X_2, and X_3 be independent standard normal random variables

$Y_1 = X_1$

$Y_2 = X_1 + X_2 + X_3$

$Y_3 = X_1 + X_3$

Obtain the

a) Joint pdf of Y_1 and Y_2 and Y_3 [7 marks]

b) Density function of Y_1 [6 marks]

QUESTION FIVE [13 MARKS]

Given that $X_1, \ldots, X_n$ is a random sample from $N(0,1)$ obtain the distribution of sample mean, $\bar{X}$, using the moment generating function technique. [13 marks]

QUESTION SIX [13 MARKS]

Let X_1 and X_2 be two independent random variables that have gamma distribution be distributed as $Gamma\ (\alpha, 1)$ and X_2 be distributed as $Gamma\ (\beta, 1)$. Let $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$.

a) Determine whether Y_1 and Y_2 are independent [10 Marks]

b) Find the distribution of Y_1 and Y_2 [3 marks]

QUESTION SEVEN [13 MARKS]

a) Let $Y = X_1 + X_2 + ... + X_{15}$ be the sum of a random sample of size 15 from the distribution whose density function is given by;

b)

$$f(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -1 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

c) Using central limit theorem calculate the approximate value of $P(0.3 \le Y \le 1.5)$ [5 Marks]

d) If $X \sim N(\mu, \sigma^2)$ obtain the characteristic function. [8 Marks]