



**ALUPE UNIVERSITY**  
**COLLEGE**

*... Bastion of Knowledge...*

P. O.Box 845-50400 Busia(K)

principal@auc.ac.ke

Tel: +254 741 217 185

+254 736 044 469

off Busia-Malaba road

**OFFICE OF THE DEPUTY PRINCIPAL  
ACADEMICS, STUDENT AFFAIRS AND RESEARCH**

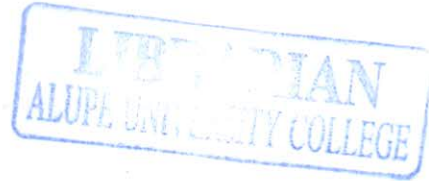
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**UNIVERSITY EXAMINATIONS**

**2019 /2020 ACADEMIC YEAR**

**SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION/ THIRD  
YEAR FIRST SEMESTER REGULAR EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH  
COMPUTING)**



**COURSE CODE: STA 216**

**COURSE TITLE: MATHEMATICAL STATISTICS II**

**DATE: 28<sup>TH</sup> OCTOBER, 2020**

**TIME: 0900 – 1200 HRS**

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**INSTRUCTION TO CANDIDATES**

**a) SEE INSIDE**

**THIS PAPER CONSISTS OF 3 PRINTED PAGES**

**PLEASE TURN OVER**

REGULAR – MAIN EXAM

## STA 216: MATHEMATICAL STATISTICS II

STREAM: ASC

DURATION: 3 hours

**INSTRUCTION TO CANDIDATES**Answer **ALL** questions from section A and any **THREE** from section B.**SECTION A [31 Marks] Answer All questions]****QUESTION ONE [15 MARKS]**

a) Define the following terms giving examples [6 marks]

- a) Joint probability density function.
- b) Stochastic independence.
- c) Marginal density.

b) If X and Y have joint p.d.f  $f(x, y)$  describe the conditional density of X given Y.

[3 Marks]

c) Let the joint p.d.f of  $x_1$  and  $x_2$  be  $f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$ 

- a) Find the marginal probability density function of  $x_1$  and  $x_2$ . [3 marks]
- b) Show that  $x_1$  and  $x_2$  are dependent. [3 marks]

**QUESTION TWO [16 MARKS]**Let X, Y, and Z have a joint pdf  $X, Y, Z \in \mathbb{R}^3$ 

$$f(X, Y, Z) = \frac{1}{K}, 0 < X < 1, 0 < Y < 1, 0 < Z < 1$$

Find the

- a) Value of K [2 marks]
- b) Marginal density functions  $f(X)$ ,  $f(Y)$  and  $f(Z)$  [6 marks]
- c) Conditional density function  $f(X/YZ)$ ,  $f(Z/XY)$  [6 marks]
- d) Expectation  $E(Y/XY)$  [2 marks]

**SECTION B [39 Marks] ANSWER ANY THREE QUESTIONS**

**QUESTION THREE [13 MARKS]**

a) Describe any three properties of a characteristic function [6 Marks]

b) Let  $X_1$  and  $X_2$  be independent, with  $X_1$  normal  $(0,1)$  and  $X_2$  chi-square with  $r$  degrees of freedom. Show that the random variable  $Y = \frac{\sqrt{r} X_1}{\sqrt{X_2}}$  has the T distribution with  $r$  degrees of freedom. [7 Marks]

**QUESTION FOUR [13 MARKS]**

Let  $X_1, X_2,$  and  $X_3$  be independent standard normal random variables

$$Y_1 = X_1$$

$$Y_2 = X_1 + X_2 + X_3$$

$$Y_3 = X_1 + X_3$$

Obtain the

a) Joint pdf of  $Y_1$  and  $Y_2$  and  $Y_3$  [7 marks]

b) Density function of  $Y_1$  [6 marks]



**QUESTION FIVE [13 MARKS]**

Given that  $X_1, \dots, X_n$  is a random sample from  $N(0,1)$  obtain the distribution of sample mean,  $\bar{X}$ , using the moment generating function technique. [13 marks]

**QUESTION SIX [13 MARKS]**

Let  $X_1$  and  $X_2$  be two independent random variables that have gamma distribution be distributed as  $Gamma(\alpha, 1)$  and  $X_2$  be distributed as  $Gamma(\beta, 1)$ . Let  $Y_1 = X_1 + X_2$  and  $Y_2 = \frac{X_1}{X_1 + X_2}$ .

a) Determine whether  $Y_1$  and  $Y_2$  are independent [10 Marks]

b) Find the distribution of  $Y_1$  and  $Y_2$  [3 marks]

**QUESTION SEVEN [13 MARKS]**

- a) Let  $Y = X_1 + X_2 + \dots + X_{15}$  be the sum of a random sample of size 15 from the distribution whose density function is given by;

$$f(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -1 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

b)

- c) Using central limit theorem calculate the approximate value of  $P(0.3 \leq Y \leq 1.5)$  [5 Marks]

- d) If  $X \sim N(\mu, \sigma^2)$  obtain the characteristic function.

[8 Marks]